PRESSURE DROP AND VELOCITY DISTRIBUTION IN THE LAMINAR ENTRANCE REGION OF A TRIANGULAR DUCT

by

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UNITED STATES NAVAL POSTGRADUATE SCHOOL

THESIS

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ABSTRACT

The developing flow in the hydrodynamic entrance region of an equilateral triangular channel was investigated. Using a fully developed velocity profile from Knudsen and Katz (3) as a boundary condition, the equations were solved numerically employing the method of Chorin (1).

The resulting velocity profiles and pressure drop were employed in calculating the local friction factor in the hydrodynamic entrance region.

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TABLE OF SYMBOLS

English Letter Symbols

a	Altitude of triangle
c	Artificial speed of sound
d	Hydraulic diameter, 4x area/perimeter
n	Number of space parameters
N	Iteration index
p	Pressure
P	Dimensionless pressure, $p\left[\frac{d}{\rho\nu u_o}\right]$
t	Artificial time parameter
t'	Dimensionless time, $t\left[\frac{\nu}{d^2}\right]$
u	Axial velocity component
u_o	Initial axial velocity component
U	Dimensionless axial velocity, u/u_o
v	Horizontal velocity component
V	Dimensionless horizontal velocity, v/u_o
w	Vertical velocity component
W	Dimensionless vertical velocity, w/u_o
X	Dimensionless axial coordinate, x/d
X_e	Dimensionless entrance length
Y	Dimensionless horizontal coordinate, y/d
Z	Dimensionless vertical coordinate, z/d

Greek Letter Symbols

ρ	Fluid density, artificial density
μ	Fluid viscosity

ν Kinematic viscosity

$\bar{\tau}$ Average wall shear stress

Non-dimensional Groups

f Friction factor, $2\bar{\tau}/\rho u^2$

R Reynolds number based on hydraulic diameter, du/ν

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I. INTRODUCTION

The flow characteristics of non-circular channels have become of increasing importance in the analysis of compact heat exchangers. Nevertheless, heat transfer analyses of the triangular entrance region have been limited to approximations which postulate fully established velocity profiles (Graetz Approximation). Such solutions have been obtained by Lu and Miller (4) and Tao (8) for equilateral triangles.

Using Sparrow's (7) results for the fully established velocity profiles in an isosceles triangular duct, McComas (5) has estimated values of entrance lengths and friction factors for such flows.

In order to complete the analysis of heat transfer in the entrance region of triangular ducts, it is necessary to obtain a hydrodynamic solution for the developing velocity profiles in that region. It is therefore the purpose of the present work to provide a solution of the equations of motion in the hydrodynamic entrance region of an equilateral triangular duct.

II. ANALYSIS

In an attempt to find a convergent solution to the equations of motion, several explicit methods were tried without success. Subsequently a relaxation method employing an artificial density change, suggested by Chorin (1), was adopted. This method essentially alters the solution of the equations of motion from an initial value problem to a boundary value problem, necessitating the use of entrance length and the fully developed velocity profile as boundary conditions.

A. Governing Equations

The governing equations for the flow of an incompressible viscous fluid (Navier Stokes Equations) may be written:

x - Momentum:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\nabla^2 u \tag{1}$$

y - Momentum:

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\nabla^2 v \tag{2}$$

z - Momentum:

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\nabla^2 w \tag{3}$$

Continuity:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{4}$$

Introducing the dimensionless variables,

$$U = \frac{u}{u_0} \qquad V = \frac{v}{u_0} \qquad W = \frac{w}{u_0}$$

$$X = \frac{x}{d} \qquad Y = \frac{y}{d} \qquad Z = \frac{z}{d}$$

$$P = p\,\frac{d}{\rho \nu u_0} \qquad t' = t\,\frac{\nu}{d^2} \tag{5}$$

equations 1-4 become:

$$\frac{\partial U}{\partial t'} + R\left[U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} + W\frac{\partial U}{\partial Z}\right] = -\frac{\partial P}{\partial X} + \nabla^2 U \tag{6}$$

$$\frac{\partial V}{\partial t'} + R\left[U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y} + W\frac{\partial V}{\partial Z}\right] = -\frac{\partial P}{\partial Y} + \nabla^2 V \tag{7}$$

$$\frac{\partial W}{\partial t'} + R\left[U\frac{\partial W}{\partial X} + V\frac{\partial W}{\partial Y} + W\frac{\partial W}{\partial Z}\right] = -\frac{\partial P}{\partial Z} + \nabla^2 W \tag{8}$$

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} + \frac{\partial W}{\partial Z} = 0 \tag{9}$$

B. Boundary Conditions

A condition of uniform velocity is assumed at the entrance and no-slip conditions are imposed at the walls; thus:

1. at the entrance:*

$$U(0,Y,Z,t) = 1$$
$$V(0,Y,Z,t) = 0$$
$$W(0,Y,Z,t) = 0$$

2. at the walls:

$$U(t) = V(t) = W(t) = 0$$

---

*The prime has been dropped and dimensionless variables are understood.

C. Artificial Density

Because the problem is one of incompressible flow, no density change can take place. One can introduce an auxiliary continuity equation,

$$\frac{\partial \rho}{\partial t} + \frac{\partial U}{\partial X} + \frac{\partial W}{\partial Z} + \frac{\partial V}{\partial Y} = 0 \tag{9a}$$

in which changes of the pseudo density given by

$$P = \rho / \delta \tag{10}$$

can be used to test for convergence at each iteration process. Thus, we have introduced a set of auxiliary parameters, ρ and δ, which are analogous to an artificial density and an artificial compressibility.

By introducing the artificial equation of state (equation 10) and using t' as an auxiliary parameter analogous to time in compressible flow, the initial value problem can be altered to a boundary value problem. Such a change, however, requires the use of a fully established solution as an additional boundary condition. An exact solution for the steady, fully established profile has been obtained by Knudsen and Katz (3):

$$U(Y,Z) = -2fR\left[\frac{1}{4}(\eta^2 + \zeta^2) - \frac{1}{6}(\eta^3 - 3\eta\zeta^2) - \frac{1}{12}\right] \tag{11}$$

where: $\eta = Y - \frac{a}{d\sqrt{3}}$ and: $\zeta = Z - \frac{a}{3d}$. The friction factor f for fully developed flow is given by Kays (2) as 13.33. This number serves to determine the value of the fully established pressure gradient.

D. Finite Difference Equations

Equations 6-9 were put into finite difference form using central differences, except for the second derivatives which were differenced using the Dufort-Frankel pattern. This latter expression is of the form:

$$\frac{\partial^2 U}{\partial X^2} \cong \frac{U^N(X+) + U^N(X-) - U^{N+1} - U^{N-1}}{\Delta X^2} \tag{12}$$

where $U^N(X+)$ denotes $U(N, X+\Delta X, Y, Z)$, U^{N+1} is $U(N+1, X, Y, Z)$ and N denotes the Nth relaxation. The resulting equations were solved for the N+1 terms and the resulting computing equations are of the form:

$$\begin{aligned} U^{N+1} = \Big\{ 1 \Big/ \Big(1 + \frac{2\Delta t}{\Delta X^2} + \frac{2\Delta t}{\Delta Y^2} + \frac{2\Delta t}{\Delta Z^2}\Big)\Big\} \times \Big\{ &-\frac{\Delta t}{\Delta X}\frac{1}{\delta}\left[\rho^N_{(X+)} - \rho^N_{(X-)}\right] \\ &- R\frac{\Delta t}{\Delta X}\left[U^N_{(X+)}U^N_{(X+)} - U^N_{(X-)}U^N_{(X-)}\right] - R\frac{\Delta t}{\Delta Y}\left[U^N_{(Y+)}V^N_{(Y+)} - U^N_{(Y-)}V^N_{(Y-)}\right] \\ &- R\frac{\Delta t}{\Delta Z}\left[U^N_{(Z+)}W^N_{(Z+)} - U^N_{(Z-)}W^N_{(Z-)}\right] + \frac{2\Delta t}{\Delta X^2}\left[U^N_{(X+)} + U^N_{(X-)} - U^{N-1}\right] \\ &+ \frac{2\Delta t}{\Delta Y^2}\left[U^N_{(Y+)} + U^N_{(Y-)} - U^{N-1}\right] + \frac{2\Delta t}{\Delta Z^2}\left[U^N_{(Z+)} + U^N_{(Z-)} - U^{N-1}\right]\Big\} \end{aligned} \tag{13}$$

$$\begin{aligned} V^{N+1} = \Big\{ 1 \Big/ \Big(1 + \frac{2\Delta t}{\Delta X^2} + \frac{2\Delta t}{\Delta Y^2} + \frac{2\Delta t}{\Delta Z^2}\Big)\Big\} \times \Big\{ &-\frac{\Delta t}{\Delta Y}\frac{1}{\delta}\left[\rho^N_{(Y+)} - \rho^N_{(Y-)}\right] \\ &- R\frac{\Delta t}{\Delta X}\left[V^N_{(X+)}U^N_{(X+)} - V^N_{(X-)}U^N_{(X-)}\right] - R\frac{\Delta t}{\Delta Y}\left[V^N_{(Y+)}V^N_{(Y+)} - V^N_{(Y-)}V^N_{(Y-)}\right] \\ &- R\frac{\Delta t}{\Delta Z}\left[V^N_{(Z+)}W^N_{(Z+)} - V^N_{(Z-)}W^N_{(Z-)}\right] + \frac{2\Delta t}{\Delta X^2}\left[V^N_{(X+)} + V^N_{(X-)} - V^{N-1}\right] \\ &+ \frac{2\Delta t}{\Delta Y^2}\left[V^N_{(Y+)} + V^N_{(Y-)} - V^{N-1}\right] + \frac{2\Delta t}{\Delta Z^2}\left[V^N_{(Z+)} + V^N_{(X-)} - V^{N-1}\right]\Big\} \end{aligned} \tag{14}$$

$$\begin{aligned} W^{N+1} = \Big\{ 1 \Big/ \Big(1 + \frac{2\Delta t}{\Delta X^2} + \frac{2\Delta t}{\Delta Y^2} + \frac{2\Delta t}{\Delta Z^2}\Big)\Big\} \times \Big\{ &-\frac{\Delta t}{\Delta Z}\frac{1}{\delta}\left[\rho^N_{(Z+)} - \rho^N_{(Z-)}\right] \\ &- R\frac{\Delta t}{\Delta X}\left[W^N_{(X+)}U^N_{(X+)} - W^N_{(X-)}U^N_{(X-)}\right] - R\frac{\Delta t}{\Delta Y}\left[W^N_{(Y+)}V^N_{(Y+)} - W^N_{(Y-)}V^N_{(Y-)}\right] \\ &- R\frac{\Delta t}{\Delta Z}\left[W^N_{(Z+)}W^N_{(Z+)} - W^N_{(Z-)}W^N_{(Z-)}\right] + \frac{2\Delta t}{\Delta X^2}\left[W^N_{(X+)} + W^N_{(X-)} - W^{N-1}\right] \\ &+ \frac{2\Delta t}{\Delta Y^2}\left[W^N_{(Y+)} + W^N_{(Y-)} - W^{N-1}\right] + \frac{2\Delta t}{\Delta Z^2}\left[W^N_{(Z+)} + W^N_{(Z-)} - W^{N-1}\right]\Big\} \end{aligned} \tag{15}$$

$$\begin{aligned} \rho^{N+1} = \rho^{N-1} &- \frac{\Delta t}{\Delta X}\left[U^N_{(X+)} - U^N_{(X-)}\right] \\ &- \frac{\Delta t}{\Delta Y}\left[V^N_{(Y+)} - V^N_{(Y-)}\right] - \frac{\Delta t}{\Delta Z}\left[W^N_{(Z+)} - W^N_{(Z-)}\right] \end{aligned} \tag{16}$$

E. Grid Size and Stability

The geometry of an equilateral triangle presents a problem in selecting appropriate values of ΔY and ΔZ. The requirement that the network intersections fall on a boundary was considered essential for numerical calculation, dictating the relationship between ΔY and ΔZ such that $\Delta Y = \sqrt{3} \times \Delta Z/2$.

The value of ΔX, while arbitrary, is constrained by the fact that the number of X-wise stations must be chosen such that the number of stations times the value of ΔX be equal to the predicted entrance length, or $\Delta X \times I = X_e$, where I is some integer. Finally, because of the limited storage available in a computer, the number of grid points is limited. For the purposes of this investigation, the values adopted were $\Delta X = .00159$, $\Delta Y = 0.17321$, and $\Delta Z = 0.3$. This grid resulted in 25 stations in the X-direction, 11 in the Y and 6 in the Z, as shown in Figure 1.

Having selected the grid size, the stability of equations 13-16 may be determined using the results of Chorin (1), who points out that the introduction of an artificial density results in an artificial speed of sound of the form:

$$C^2 = 1/\delta \tag{17}$$

A Mach number relative to the speed of sound is given by,

$$M = \frac{R}{C}\left(\bar{U}^2_{MAX} + \bar{V}^2_{MAX} + \bar{W}^2_{MAX}\right)^{1/2} \tag{18}$$

For convergence it is necessary that the Mach number (M) be less than one. With this condition satisfied, the system of equations is stable when:

$$\Delta t \leq \frac{2}{n^{1/2}\,(1+5^{1/2})} \times \left(\text{MINIMUM DIMENSION}\right)^{1/2} \tag{19}$$

In the present work these values were taken to be M=0.975, δ=9.38025, Δt=0.00035, and R=1.0.

F. Friction Factor

The wall shear stress may be related to the local pressure gradient, which is of importance to engineers in predicting pressure drop. The average stress at any section is given by the Newtonian shearing law,

$$\bar{\tau} = \mu\left(\frac{\overline{\partial u}}{\partial n}\right)_B \tag{20}$$

where $\left(\frac{\overline{\partial u}}{\partial n}\right)_B$ represents the average velocity gradient evaluated at the wall.

Using equation 20, a dimensionless friction factor can be defined,

$$f = \frac{\bar{\tau}}{\frac{1}{2}\rho u^2} \tag{21}$$

in which f is the usual flow friction factor.

G. Numerical Solution

In the present analysis, using the values of ΔX, ΔY, ΔZ, Δt and δ given above, equations 13-16 were found to converge to a steady solution after 2,000 iterations. While Chorin points out that initial values of variables may be set at zero throughout, the convergence can be hastened by using a good "guess" at the values to be calculated. In the present case the fully established velocity values were used throughout as an initial assumption.

A copy of the program written in Fortran IV language is given in the Appendix.

III. RESULTS

The dimensionless centerplane velocity profiles are given in Figure 2 for five values of X/X_e (0.2, 0.4, 0.6, 0.8, and 1.0). Figures 3 through 7 show the centerplane velocity profiles and velocity contours for X/X_e=0.2, 0.4, 0.6, 0.8, and 1.0 respectively. The secondary velocities, V and W, are shown in Figure 8 at a station about midway down the entrance region at X/X_e=0.6. The value of friction factor at each x-wise station is plotted in Figure 9 in the form of f/f_e versus X/X_e, where the e subscript denotes the fully established values.

The dashed line on the friction factor curve was computed from the local skin friction predicted from a boundary layer analysis. The value of skin friction must of course be unbounded at X=0 because of the discontinuity in velocity at the lip of the duct. For large values of X/X_e, f/f_e must approach unity asymptotically as the results indicate.

IV. CONCLUSIONS

From the results of this study, it may be concluded that:

1. The relaxation procedure used in this analysis was found to be convergent subject to certain stability parameters as Chorin (1) points out. The results, however, remain unchecked experimentally.

2. The use of the fully established values as exit boundary conditions forces the relaxation values to converge on these values in the limit as the number of iterations increase. Therefore, the values of the computed velocity profiles can be only as accurate as the exit boundary conditions. Due to the lack of analytic and experimental results in the area, there was little choice in using the results of Knudsen and Katz (3) for the fully established velocity profile and Kays (5) for the fully established pressure gradient.

3. In general, the grid sizes used should be reduced until the number of points reaches the limit of computer storage. This serves to maximize the accuracy of the results consistent with the accuracy of the fully established profiles used.

4. It seemed intuitively obvious when first using Chorin's method that because it represents a convergent numerical solution to the finite difference form of the Navier Stokes and continuity equations, using an entrance length greater than a predicted value would lead to the fully established profile upstream of the boundary condition. This, however, was not the case. Thus, the necessity of using the predicted value of X_e reduces the generality of this method in the solution of physical problems. The use of a Reynolds number of unity may have been the cause of this, however it would seem evident that a

complete analysis of the equations must be made numerically to find the reason for this limitation.

5. The Reynolds number dependency of the governing equations can be extracted by using non-dimensional space parameters of the form $X=xR/d$. An analysis of the resulting equations would seem beneficial as the results would be completely general and would represent "universal" profiles.

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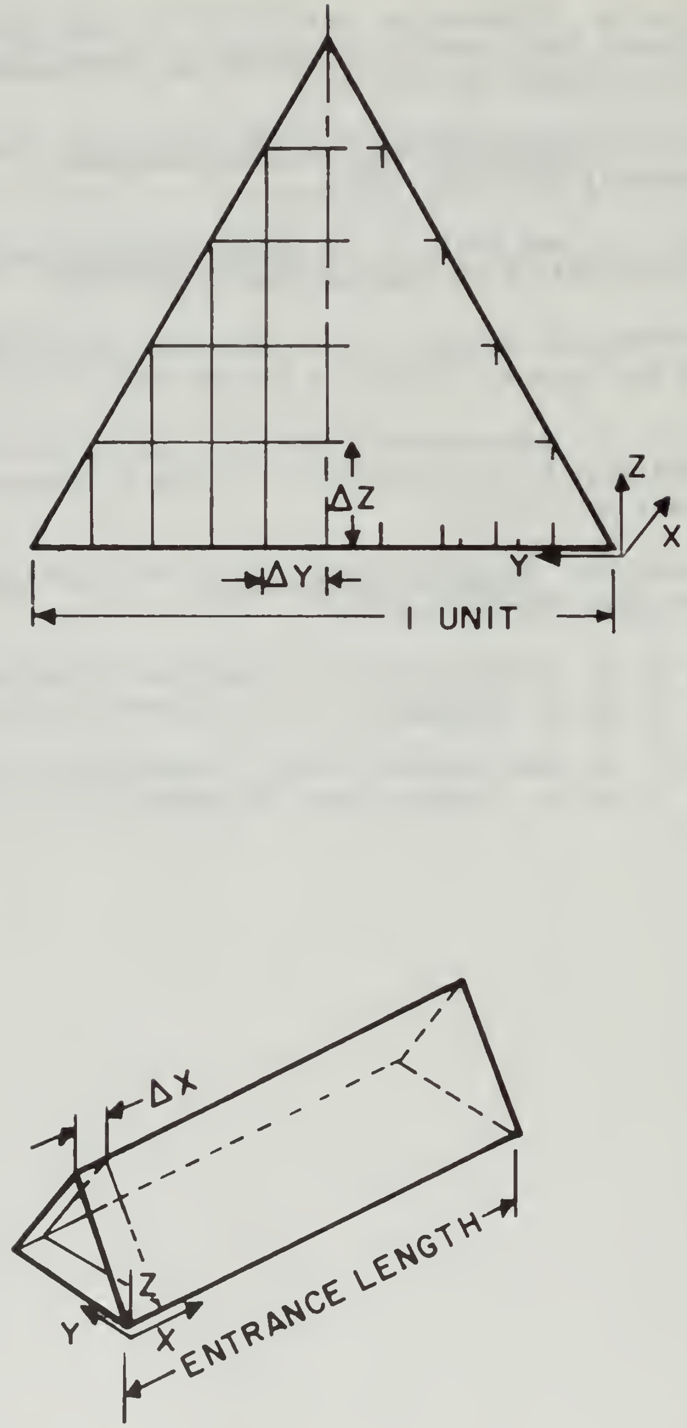

FIGURE I.

AXES ORIENTATION
AND MESH POINTS.

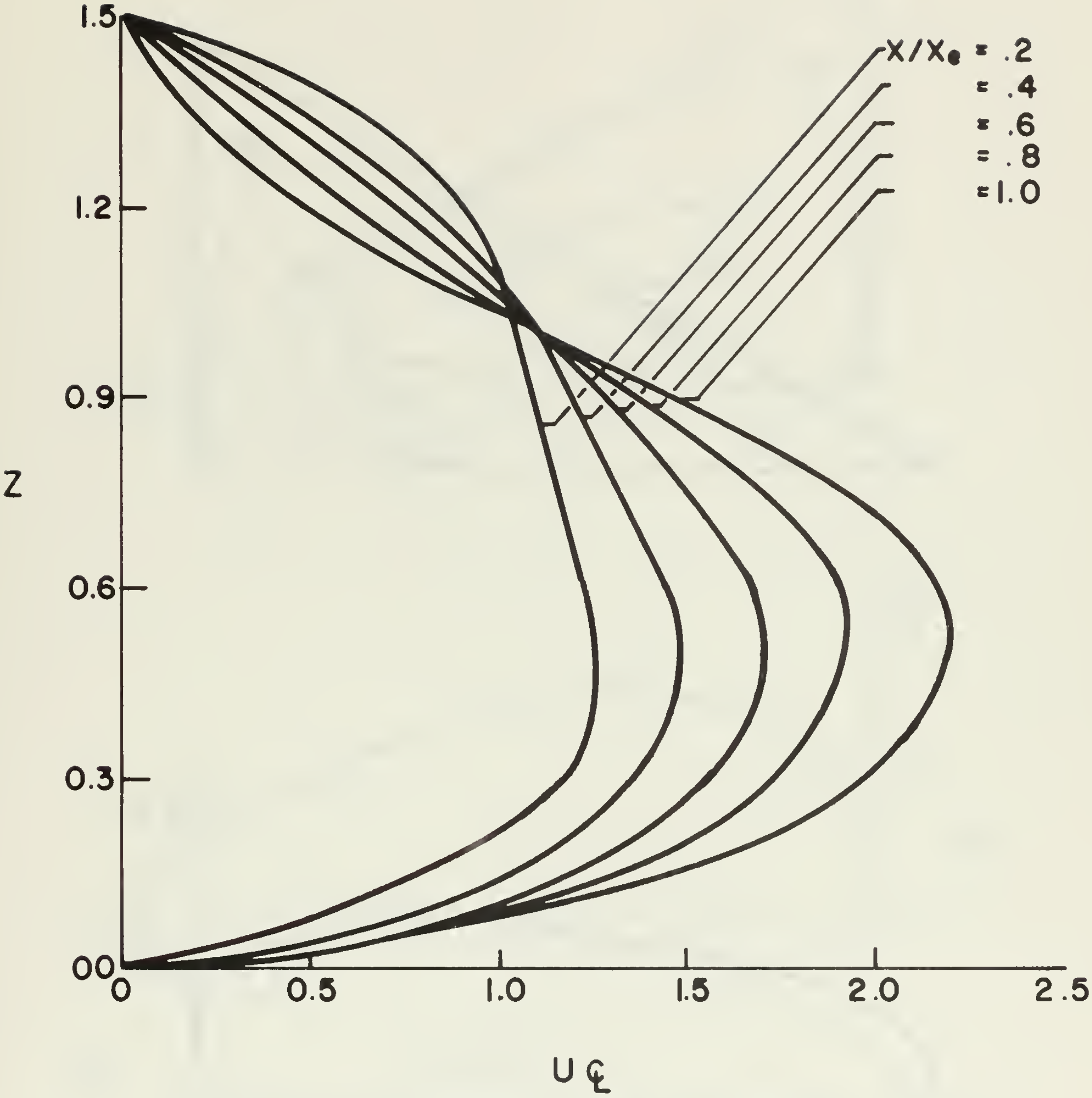

FIGURE 2.

DIMENSIONLESS VELOCITY PROFILES FOR VALUES OF X/X$_e$.

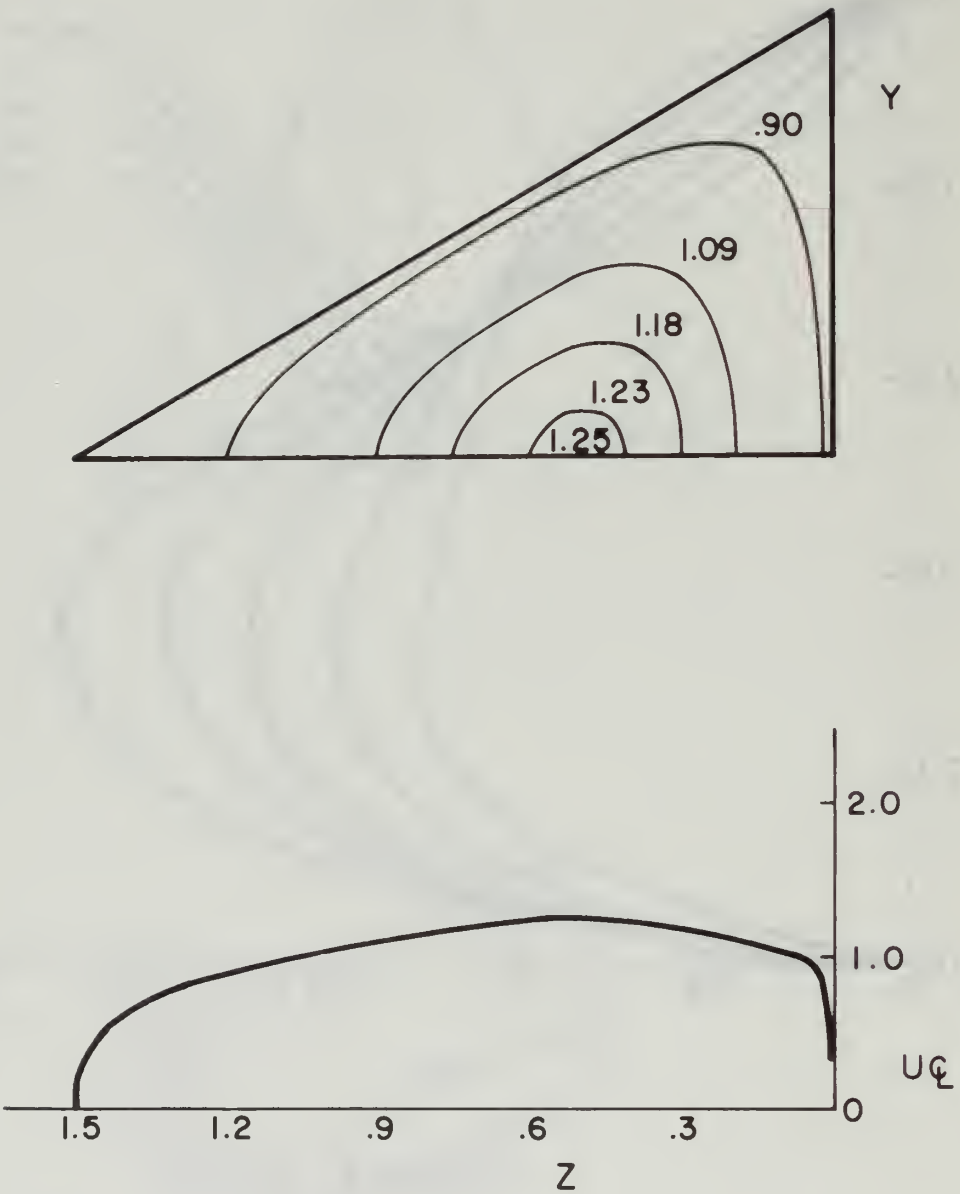

FIGURE 3.

VELOCITY PROFILE AND CONTOURS FOR $X/X_e = 0.2$

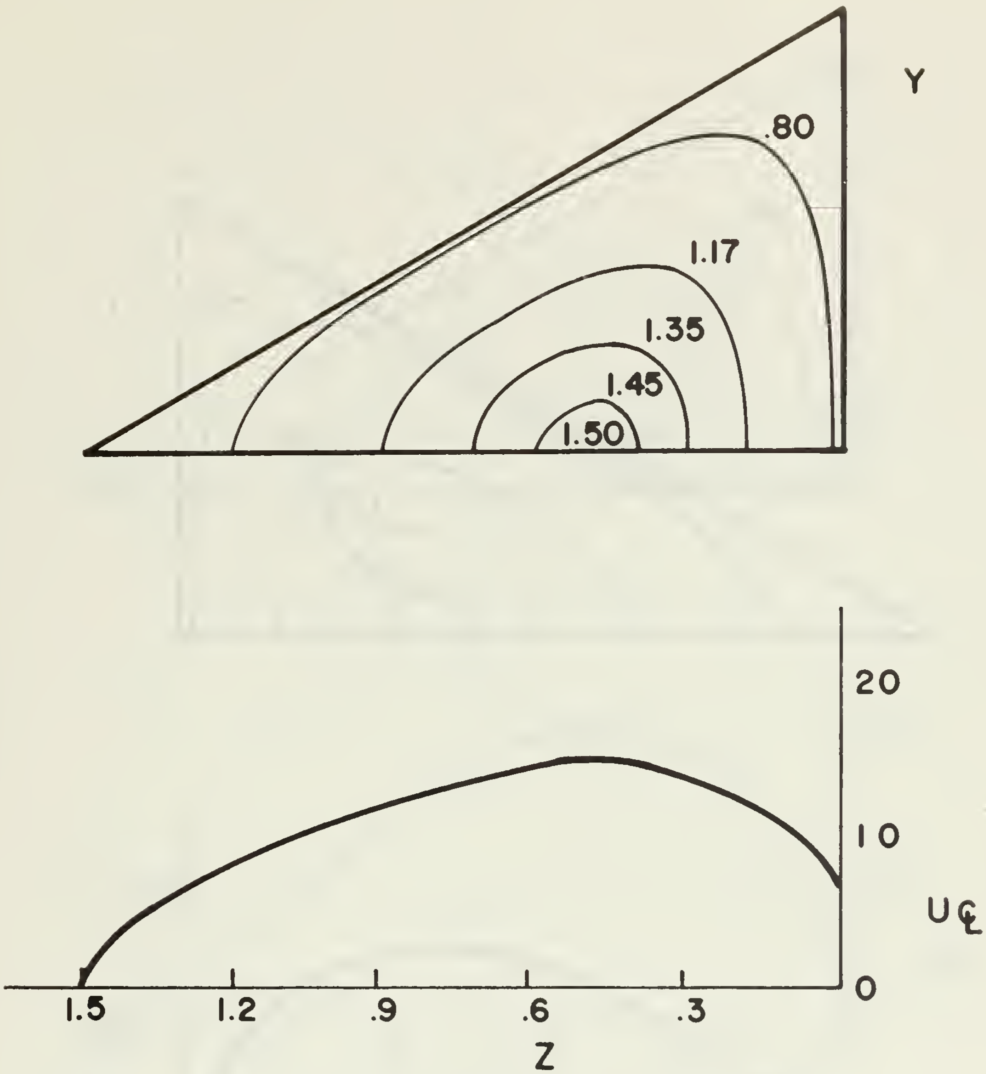

FIGURE 4.

VELOCITY PROFILE AND CONTOURS FOR $X/X_e = 0.4$

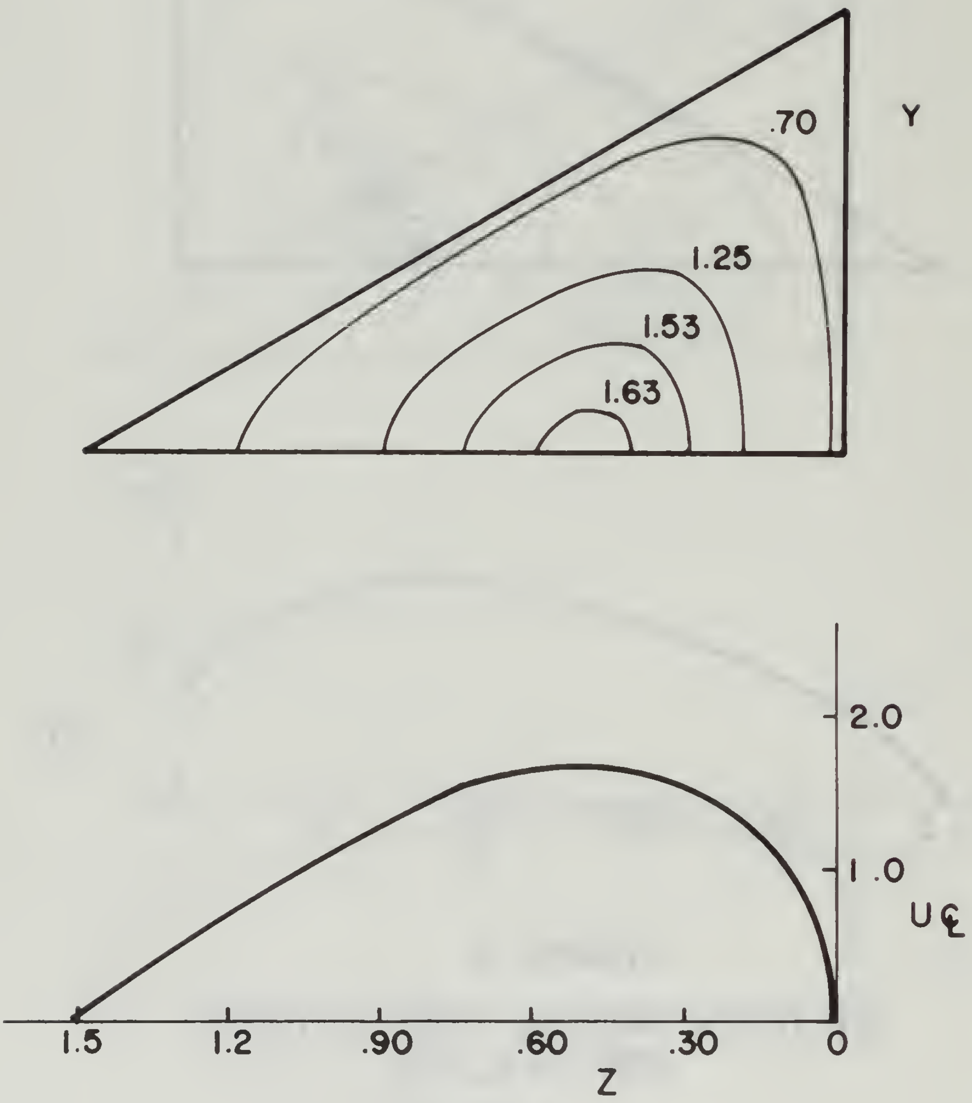

FIGURE 5.

VELOCITY PROFILE AND CONTOURS FOR $X/X_e = 0.5$

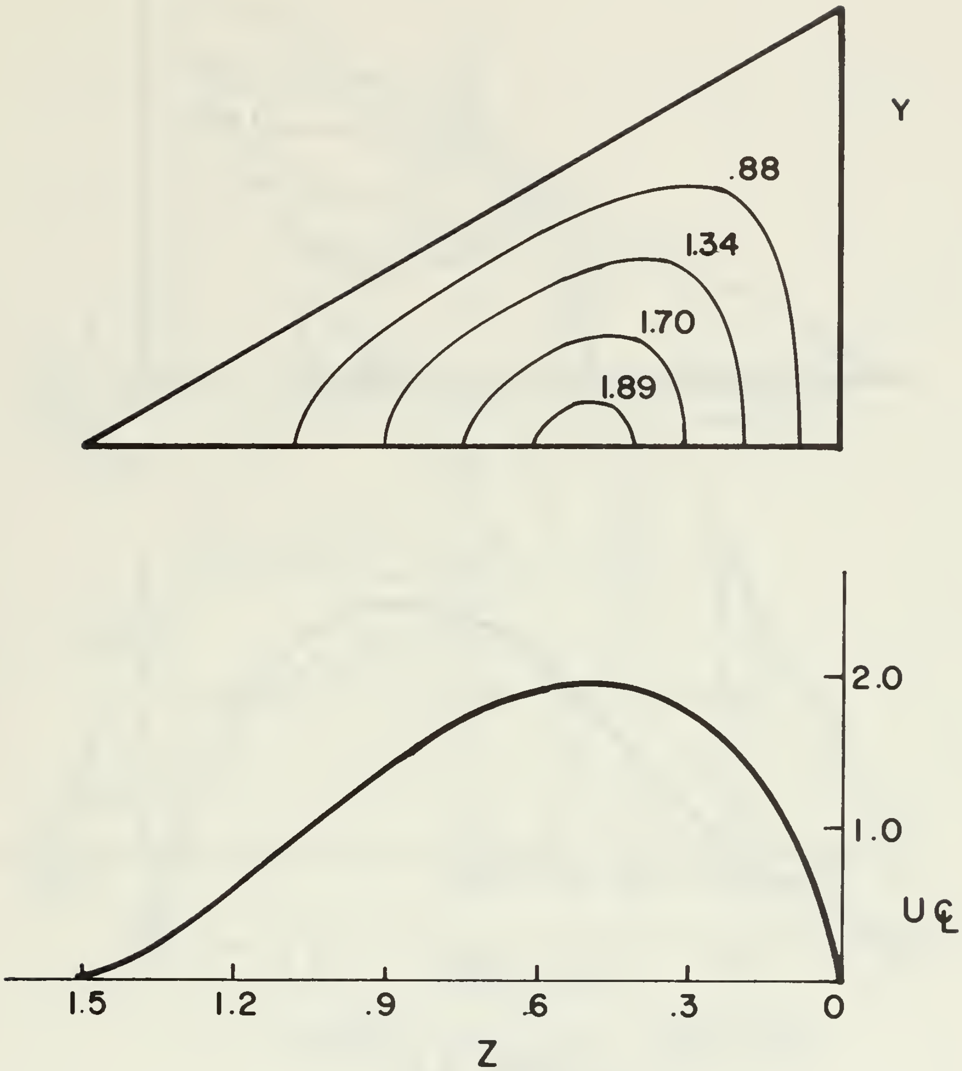

FIGURE 6.

VELOCITY PROFILE AND CONTOURS FOR $X/X_e = 0.8$

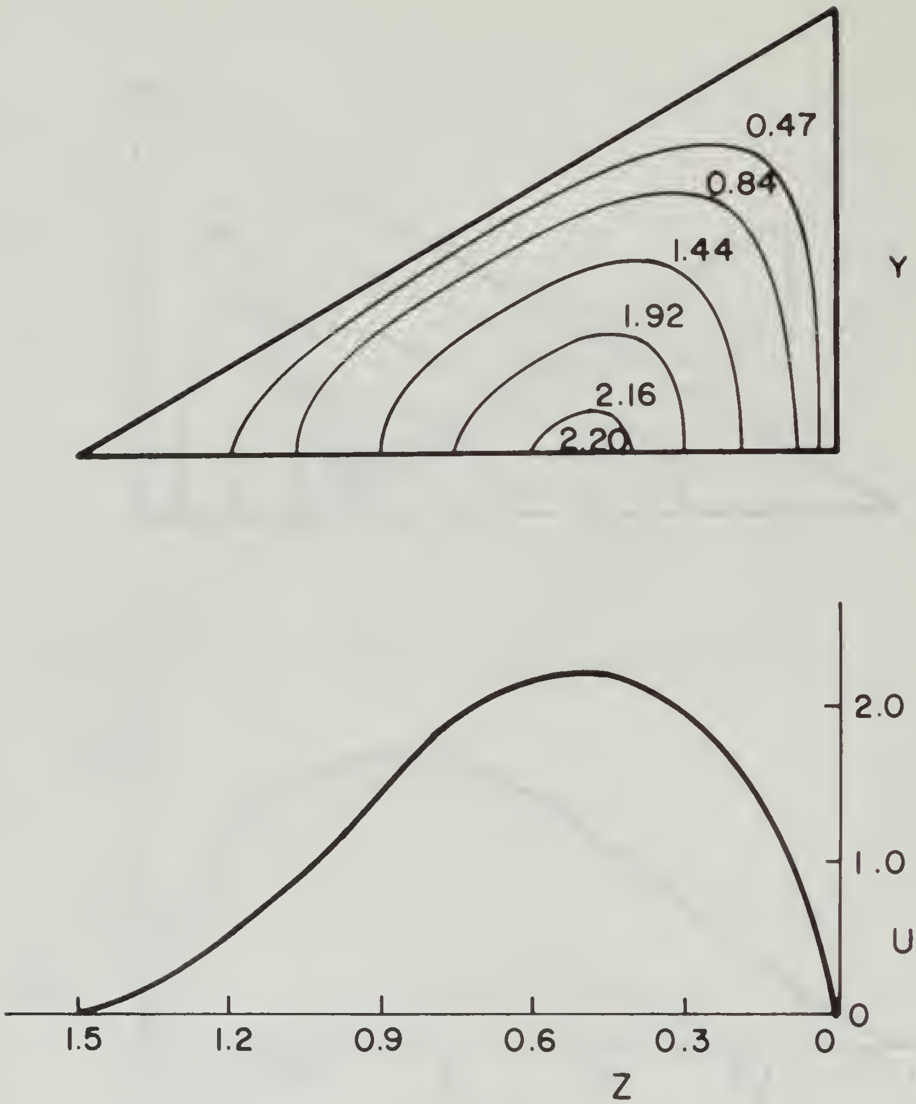

FIGURE 7.

VELOCITY PROFILE AND CONTOURS
FOR $X/X_e = 1.0$

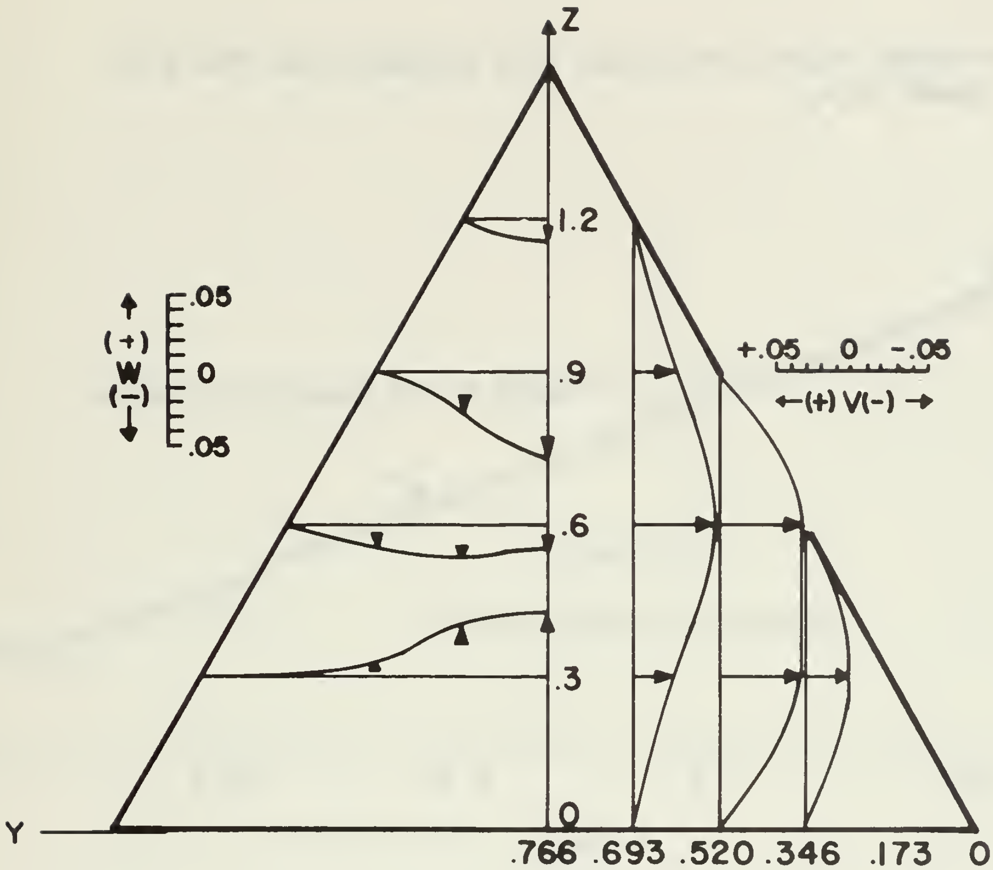

FIGURE 8.

SECOND VELOCITY PROFILES
FOR $X/X_e = 0.6$

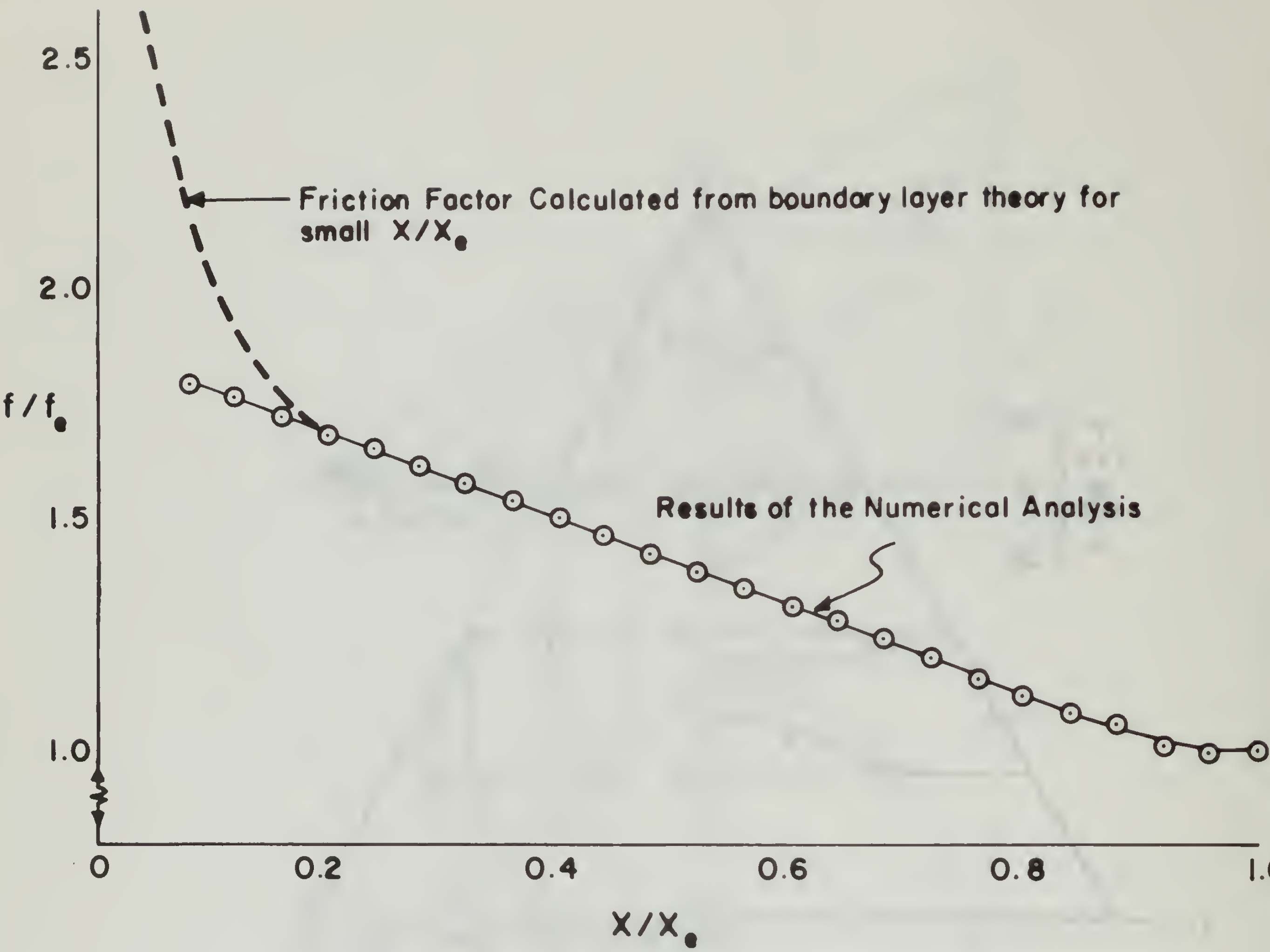

FIGURE 9.
FRICTION FACTOR RATIO vs. X/X_e

APPENDIX

SAMPLE FORTRAN IV PROGRAM

```
C     PROGRAM TO EVALUATE CHANNEL FLOW USING RELAXATION
      DIMENSION U(3,25,11,6)
      DIMENSION V(3,25,11,6)
      DIMENSION W(3,25,11,6)
      DIMENSION P(3,25,11,6)
      CALL CANCEL (2)
C     P IS THE ARTIFICAL COMPRESSIBILITY TERM (3OF)
      READ (5,1) NM,MAX,INC
 1    FORMAT(3I10)
      READ (5,2) RE,DDX,DPDX
 2    FORMAT(3F20.10)
      READ (5,6) SMACH
 6    FORMAT(F20.10)
      NUM=NM+1
      NUMK=NM/2.+1
      DS=2./NM
      DY=DS/2.
      ALPHA=DS**2-DY**2
      DZ=SQRT(ALPHA)
      D=SQRT(3.)/2.
      DZ=DZ/D
      DY=DY/D
      DX=DDX/D
      UU=2.5
      FAC=2./(SQRT(3.)*(1.+SQRT(5.)))
      C=(RE/SMACH)*SQRT(UU)
      DEL=1./C**2
      DT=FAC*DX*SQRT(DEL)
      INCX=INC-1
C     ZEROING ALL STORAGE SPACES
      DO 3 N=1,3
      DO 3 I=1,INC
      DO 3 J=1,NUM
      DO 3 K=1,NUMK
      U(N,I,J,K)=0.0
      V(N,I,J,K)= .0
      W(N,I,J,K)= .0
      P(N,I,J,K)=0.0
 3    CONTINUE
C     CONSTANTS
      C1=RE*DT/DX
      C2=RE*DT/DY
      C3=RE*DT/DZ
      S1=2.*DT/DX**2
      S2=2.*DT/DY**2
      S3=2.*DT/DZ**2
      SUM=1+S1+S2+S3
      SSUM=1+    S2+S3
      E1=DT/(DX*DEL)
      E2=DT/(DY*DEL)
      E3=DT/(DZ*DEL)
C     BOUNDRY CONDITIONS AT EXIT
      WRITE (6,9 ) RE,DX,DY,DZ,DT,DEL,DPDX,INC,MAX
 9    FORMAT(1H1///T5,'RE',T15,'DX',T25,'DY',T35,'DZ',T45,'DT',T55,
     1'DEL',T65,'DPDX',T75,'INC',T89,'MAX',//7E10.5,2(5X,I5)///)
      WRITE (6,91)
 91   FORMAT(T5,'C1',T15,'C2',T25,'C3',T35,'S1',T45,'S2',T55,'S3',T65,
     1'SUM',T75,'SSUM',T85,'E1',T95,'E2',T105,'E3'/)
      WRITE (6,92) C1,C2,C3,S1,S2,S3,SUM,SSUM,E1,E2,E3
 92   FORMAT(11E10.5)
      WRITE(6,105) SMACH
 105  FORMAT(//T5,'THE MACH NUMBER IS:',E10.5)
C     BOUNDRY CONDITIONS AT EXIT
      JSUM=0.0
      A=SQRT(3.)/2.
      DO 10 J=1,NUM
      DO 10 K=1,NUMK
      Y=0.0
      Z=0.0
      LIM=K+J
      IF(K.EQ.1) GO TO 1
      IF(LIM.GE.NUM+1) GO TO 1
      IF(K.GE.J) GO TO 10
```

```
      Y=Y+DY*(J-1)
      Z=Z+DZ*(K-1)
      YR=Y-3./(2.*SQRT(3.))
      ZR=Z-1./2.
      U(3,INC,J,K)=-DPDX * ((1./4.) * (ZR**2+YR**2)
     1  -1./6. * (ZR**3 - 3.*ZR*YR**2) - 1./12.)
      USUM=USUM+U(3,INC,J,K)
1     CONTINUE
11    CONTINUE
      WRITE(6,101) (J, J=1,NUM)
101   FORMAT(//7X,I5,10I10/)
      DO 102 K=1,NUMK
      WRITE (6,103) K,(U(3,INC,J,K),J=1,NUM)
103   FORMAT (I5,11F10.5)
102   CONTINUE
      WRITE(6,76) USUM
      DO 12 N=1,3
      DO 12 J=1,NUM
      DO 12 K=1,NUMK
      I=INC
      U(2,I,J,K) = U(3,I,J,K)
      U(1,I,J,K) = U(2,I,J,K)
12    CONTINUE
      DO 4 N=1,3
      DO 4 I=2,INCX
      DO 4 J=1,NUM
      DO 4 K=1,NUMK
      LIM=K+J
      IF(K.EQ.1) GO TO 5
      IF(LIM.GE.NUM+1) GO TO 5
      IF(K.GE.J) GO TO 5
      P(N,I,J,K)=1.0
      U(N,I,J,K)=U(N,INC,J,K)
      V(N,I,J,K)= .0
      W(N,I,J,K)= .0
5     CONTINUE
4     CONTINUE
C     INITIAL CONDITIONS
      DO 14 N=1,3
      DO 14 J=1,NUM
      DO 14 K=1,NUMK
      LIM=K+J
      IF(LIM.GT.NUM+1) GO TO 14
      IF(K.GT.J) GO TO 14
      U(N,1,J,K)=1.0
      V(N,1,J,K)=0.0
      W(N,1,J,K)=0.
      P(N,1,J,K)=1.
14    CONTINUE
C     RELAXATION EQUATIONS
      XM=0
      DO 50 IT=1,MAX
      DO 50 I=2,INCX
      DO 50 J=1,NUM
      DO 50 K=1,NUMK
      LIM=K+J
      IF(K.GT.J)          GO TO 5
      IF(LIM.GT.NUM+1)    GO TO 50
      IF(K.EQ.NUMK)       GO TO 45
      IF(J.EQ.NUM)        GO TO 46
      IF(K.EQ.1)          GO TO 47
      IF(K.EQ.J)          GO TO 48
      IF(LIM.EQ.NUM+1)    GO TO 49
      P(3,I,J,K) = P(1,I,J,K)- DT/DX * (U(2,I+1,J,K)-U(2,I-1,J,K))
     1 - DT/DY * (V(2,I,J+1,K)-V(2,I,J-1,K))
     2 - DT/DZ * (W(2,I,J,K+1)-W(2,I,J,K-1))
      U(3,I,J,K) = 1./SUM * (U(1,I,J,K)
     1 - C1 * (U(2,I+1,J,K)*U(2,I+1,J,K)-U(2,I-1,J,K)*U(2,I-1,J,K))
     2 - C2 * (U(2,I,J+1,K)*V(2,I,J+1,K)-U(2,I,J-1,K)*V(2,I,J-1,K))
     3 - C3 * (U(2,I,J,K+1)*W(2,I,J,K+1)-U(2,I,J,K-1)*W(2,I,J,K-1))
     4 + S1 * (U(2,I+1,J,K)+U(2,I-1,J,K)-U(1,I,J,K))
     5 + S2 * (U(2,I,J+1,K)+U(2,I,J-1,K)-U(1,I,J,K))
     6 + S3 * (U(2,I,J,K+1)+U(2,I,J,K-1)-U(1,I,J,K))
```

```
      7 - F1 *  (P(2,I+1,J,K)-P(2,I-1,J,K)))
       V(3,I,J,K) = 1./SUM * (V(1,I,J,K)
      1 - C1 *  (V(2,I+1,J,K)*U(2,I+1,J,K)-V(2,I-1,J,K)*U(2,I-1,J,K))
      2 - C2 *  (V(2,I,J+1,K)*V(2,I,J+1,K)-V(2,I,J-1,K)*V(2,I,J-1,K))
      3 - C3 *  (V(2,I,J,K+1)*W(2,I,J,K+1)-V(2,I,J,K-1)*W(2,I,J,K-1))
      4 + S1 *  (V(2,I+1,J,K)+V(2,I-1,J,K)-V(1,I,J,K))
      5 + S2 *  (V(2,I,J+1,K)+V(2,I,J-1,K)-V(1,I,J,K))
      6 + S3 *  (V(2,I,J,K+1)+V(2,I,J,K-1)-V(1,I,J,K))
      7 - F2 *  (P(2,I,J+1,K)-P(2,I,J-1,K)))
       W(3,I,J,K) = 1./SUM * (W(1,I,J,K)
      1 - C1 *  (W(2,I+1,J,K)*U(2,I+1,J,K)-W(2,I-1,J,K)*U(2,I-1,J,K))
      2 - C2 *  (W(2,I,J+1,K)*V(2,I,J+1,K)-W(2,I,J-1,K)*V(2,I,J-1,K))
      3 - C3 *  (W(2,I,J,K+1)*W(2,I,J,K+1)-W(2,I,J,K-1)*W(2,I,J,K-1))
      4 + S1 *  (W(2,I+1,J,K)+W(2,I-1,J,K)-W(1,I,J,K))
      5 + S2 *  (W(2,I,J+1,K)+W(2,I,J-1,K)-W(1,I,J,K))
      6 + S3 *  (W(2,I,J,K+1)+W(2,I,J,K-1)-W(1,I,J,K))
      7 - F3 *  (P(2,I,J,K+1)-P(2,I,J,K-1)))
       GO TO 50
 45    CONTINUE
       U(3,I,J,K) = 0.
       V(3,I,J,K) = 0.
       W(3,I,J,K) = 0.
       P(3,I,J,K) = P(1,I,J,K) + DT/DZ*3.*W(2,I,J,K-1)
       GO TO 50
 46    CONTINUE
       U(3,I,J,K) = 0.
       V(3,I,J,K) = 0.
       W(3,I,J,K) = 0.
       P(3,I,J,K) = 0.
       GO TO 50
 47    CONTINUE
       U(3,I,J,K) = 0.
       V(3,I,J,K) = 0.
       W(3,I,J,K) = 0.
       P(3,I,J,K) = P(1,I,J,K)- DT/DX * (U(2,I+1,J,K)-U(2,I-1,J,K))
      1 - DT/DY * (V(2,I,J+1,K)-V(2,I,J-1,K))
      2 + DT/DZ * (W(2,I,J,K+1)-W(1,I,J,K))
      3-4.*DT/DZ* (W(2,I,J,K+1)-W(3,I,J,K))
       GO TO 50
 48    CONTINUE
       U(3,I,J,K) = 0.
       V(3,I,J,K) = 0.
       W(3,I,J,K) = 0.
       P(3,I,J,K) = P(1,I,J,K)- DT/DX * (U(2,I+1,J,K)-U(2,I-1,J,K))
      1 + DT/DY * (V(2,I,J+1,K)-V(1,I,J,K))
      2-4.*DT/DY* (V(2,I,J+1,K)-V(3,I,J,K))
      3 + DT/DZ * (W(1,I,J,K)-W(2,I,J,K-1))
      4-4.*DT/DZ* (W(3,I,J,K)-W(2,I,J,K-1))
       GO TO 50
 49    CONTINUE
       U(3,I,J,K) = 0.
       V(3,I,J,K) = 0.
       W(3,I,J,K) = 0.
       P(3,I,J,K) = P(1,I,J,K)- DT/DX * (U(2,I+1,J,K)-U(2,I-1,J,K))
      1 + DT/DY * (V(1,I,J,K)-V(2,I,J-1,K))
      2-4.*DT/DY* (V(3,I,J,K)-V(2,I,J-1,K))
      3 + DT/DZ * (W(1,I,J,K)-W(2,I,J,K-1))
      4-4.*DT/DZ* (W(3,I,J,K)-W(2,I,J,K-1))
       GO TO 50
 50    CONTINUE
       DO 59 I=2,INCX
       DO 59 J=1,NUM
       DO 59 K=1,NUMK
       LIM=K+J
       IF(LIM.GT.NUM+1) GO TO 59
       P(1,I,J,K) = P(2,I,J,K)
       U(1,I,J,K) = U(2,I,J,K)
       V(1,I,J,K) = V(2,I,J,K)
       W(1,I,J,K) = W(2,I,J,K)
       P(2,I,J,K) = P(3,I,J,K)
       U(2,I,J,K) = U(3,I,J,K)
       V(2,I,J,K) = V(3,I,J,K)
       W(2,I,J,K) = W(3,I,J,K)
```

```
59    CONTINUE
      MM=MM+1
      IF(MM.EQ.500) GO TO 51
      GO TO 60
51    CONTINUE
      DO 73 I=5,INC,5
      WRITE(7,72) I,(U(3,I,6,K),K=1,6),D7
72    FORMAT (I6,7F10.5)
73    CONTINUE
      WRITE (6,56) IT
56    FORMAT(1H1T5,' THE RESULTS FOR THE ',I6,' TH  ITERATION ARE AS FOL
     1LOWS:'//)
      DO 55 I=5,25,5
      IF(I.EQ.5) WRITE(6,61) I
      IF(I.GT.5) WRITE(6,71) I
61    FORMAT(T5,'FOR THE ',I4,'TH  STATION:'/)
71    FORMAT(1H1////T5,'FOR THE ',I4,'TH  STATION:'/)
      WRITE(6,62)
62    FORMAT(T5,'THE CENTERLINE PAREMETERS ARE:'//T8,'I',T12,'J',T16,'K'
     1,T34,'U',T52,'V',T70,'W',T88,'ROE'/)
      DO 52 K=1,NUMK
      J=6
      WRITE (6,53) IT,I,J,K,U(3,I,J,K),V(3,I,J,K),W(3,I,J,K),R(3,I,J,K)
53    FORMAT(I6,3I4,4E16.5)
52    CONTINUE
      WRITE(6,64)
64    FORMAT(//T5,'VELOCITY IN THE X-DIRECTION:'/)
      WRITE (6,101) (J, J=1,NUM)
      DO 54 K=1,NUMK
      WRITE (6,103) K, (U(3, I,J,K), J=1,NUM)
54    CONTINUE
      WRITE(6,65)
65    FORMAT(//T5,'VELOCITY IN THE Y-DIRECTION:'/)
      WRITE(6,101) (J, J=1,NUM)
      DO 66 K=1,NUMK
      WRITE (6,103) K, (V(3, I,J,K), J=1,NUM)
66    CONTINUE
      WRITE(6,67)
67    FORMAT(//T5,'VELOCITY IN THE Z-DIRECTION:'/)
      WRITE (6,101) (J, J=1,NUM)
      DO 68 K=1,NUMK
      WRITE (6,103) K, (W(3, I,J,K), J=1,NUM)
68    CONTINUE
      WRITE(6,69)
69    FORMAT(//T5,'ARTIFICIAL DENSITY:'/)
      WRITE (6,101) (J, J=1,NUM)
      DO 70 K=1,NUMK
      WRITE (6,103) K, (R(3, I,J,K), J=1,NUM)
70    CONTINUE
      USUM=0.0
      DO 75 J=1,NUM
      DO 75 K=1,NUMK
      USUM=USUM+U(3,I,J,K)
75    CONTINUE
      WRITE (6,76) USUM
76    FORMAT(//T5,'THE MASS FLOW RATE AT THIS STATION IS: ',F10.5/)
      DIST=DX*I/D
      WRITE(6,77) DIST
77    FORMAT(T5,'THE DISTANCE DOWN THE CHANNEL IS:',F10.5)
55    CONTINUE
      MM=0
60    CONTINUE
      WRITE (6,56) IT
      DO 80 I=1,INC
      IF(I.EQ.1) WRITE(6,61) I
      IF(I.GT.1) WRITE(6,71) I
      WRITE(6,64)
      WRITE (6,101) (J, J=1,NUM)
      DO 84 K=1,NUMK
      WRITE (6,103) K, (U(3, I,J,K), J=1,NUM)
84    CONTINUE
      WRITE(6,65)
      WRITE(6,101) (J, J=1,NUM)
```

```
      DO 86 K=1,NUMK
      WRITE (6,103) K, (V(3, I,J,K), J=1,NUM)
86    CONTINUE
      WRITE(6,67)
      WRITE (6,101) (J, J=1,NUM)
      DO 88 K=1,NUMK
      WRITE (6,103) K, (W(3, I,J,K), J=1,NUM)
88    CONTINUE
      WRITE(6,69)
      WRITE (6,101) (J, J=1,NUM)
      DO 93 K=1,NUMK
      WRITE (6,103) K, (P(3, I,J,K), J=1,NUM)
93    CONTINUE
      USUM= 0.
      DO 92 J=1,NUM
      DO 92 K=1,NUMK
      USUM=USUM+U(3,I,J,K)
92    CONTINUE
      WRITE (6,75) USUM
      IF(I.EQ.1.OR.I.EQ.INC) GO TO 96
      J=6
      K=3
      DP=U(3,I,J,K)*(U(3,I+1,J,K)-U(3,I-1,J,K))/(2.*DX)
      WRITE(6,95) DP
95    FORMAT(I5,'THE PRESSURE GRADIENT IS:',F10.5/)
96    CONTINUE
      DIST=DX*I
      WRITE(6,77) DIST
8     CONTINUE
      RETURN
      END
```

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13. ABSTRACT

The developing flow in the hydrodynamic entrance region of an equilateral triangular channel was investigated. Using a fully developed velocity profile from Knudsen and Katz (3) as a boundary condition, the equations were solved numerically employing the method of Chorin (1).

The resulting velocity profiles and pressure drop were employed in calculating the local friction factor in the hydrodynamic entrance region.

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